T_k, temperature of the disperse phase; λ, coefficient of heat conduction; D, diffusion coefficient; c_k, concentration on the drop surface; c_p, specific heat; c_∞, substance concentration in a medium.

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MODEL OF THE FLOW IN A CIRCULAR JET DEVELOPING IN A CROSS STREAM. SOLUTION OF THE INITIAL LENGTH PROBLEM

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UDC 532.517.4

A flow model and an integral method of calculating the initial length of a circular jet in a cross stream are proposed. The results of calculating the characteristics of the initial length are in satisfactory agreement with the experimental data.

As is known (see [1-7]), the flow in a jet issuing at an angle to the main stream has a complex three-dimensional character and differs significantly from the flow in a submerged jet or a jet in a cocurrent stream. The jet in a cross stream is distinguished from ordinary jet flows by the presence of a flow velocity component normal to its trajectory. Obviously, it is this velocity component that must be considered "responsible" for the particular characteristics of the flow over the jet and the jet's development. Thus, the complex pattern of pressure distribution on the underlying surface around the jet, which to some extent is qualitatively similar to the pressure distribution around a cylinder, depends, for a given jet velocity, on the cross stream velocity component normal to the jet trajectory, the underpressure beyond the jet and along its sides being proportional to the velocity head created by this velocity component. This results in radial pressure differences which must obviously lead to the appearance of secondary flows in cross sections of the jet which, in their turn, cause the experimentally observed deformation of the cross section.

The mechanism of ejection of fluid by the jet from the surrounding space must also be more complex than in ordinary jets. On the one hand, the jet develops as in a cocurrent flow, and in accordance with the hypothesis of plane sections the cocurrent flow velocity may be assumed to be equal to the component of the cross stream velocity in the direction of the jet trajectory. The mass ejected by a jet in a cocurrent flow is known to be proportional to the difference of the jet and cocurrent flow velocities. On the other hand, the flow normal to the jet trajectory must contribute to the mass added to the jet, since for this flow the jet is, as it were, a fluid-filled space and at the edge of the jet an additional mixing zone must be formed. The mass entering this mixing zone from the cross stream is entrained by the jet, from which it acquires a longitudinal momentum, and becomes, as it were, part of the jet. This mixing zone increases from the plane of symmetry towards the sides of the jet and thereby causes its lateral thickening.

Since as the cross stream velocity increases the additional ejection into the jet becomes more intense, while the length of the part of the jet on which the additional ejection takes place decreases, at a certain value of the jet/flow velocity ratio there must

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Sergo Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 6, pp. 905-912, June, 1988. Original article submitted April 21, 1987.

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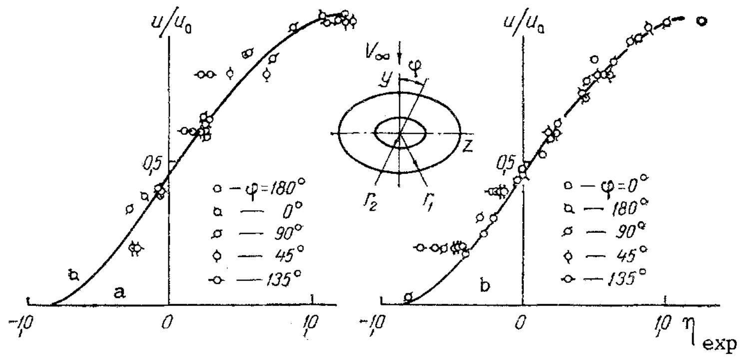

Fig. 1. Velocity profiles in cross sections of the jet: m = 0.05, x/r_0 = 2.0 (a) and 4.04 (b).

exist an optimum of the ejection capacity of the jet in the cross stream, as demonstrated experimentally in [6] and]7]. As a result of the additional ejection the rate of decrease of axial velocity should increase with increase in cross stream velocity.

With distance from the nozzle exit the longitudinal component of the cross stream velocity increases, while the component at right angles to the jet trajectory decreases. Accordingly, with distance from the nozzle exit the rate of ejection of fluid into the jet from the surrounding space should decrease.

Below, on the basis of the flow pattern described and the above considerations concerning the mechanism of interaction of the jet and the cross stream we propose a model of the flow in a circular jet developing in a cross stream and a solution, using the integral method, of the problem of the initial length of such a jet. We note that there are many studies (see [8]) in which various approaches to the determination of the characteristics of a circular jet in a cross stream are developed. However, it has not yet proved possible to obtain satisfactory agreement between the calculated and experimental data on the velocity profiles and the jet boundaries, and the literature offers no solution of the problem of the initial length of a circular jet in a cross stream.

We will assume that as a result of the secondary flows the jet is somewhat compressed in the plane of symmetry as compared with a jet in a cocurrent flow, while in the lateral direction it expands, the rate at which the additional change in the jet boundaries proceeds being proportional to the cross stream velocity component normal to the jet trajectory:

$$v_a = \varepsilon V_{\infty n}. \tag{1}$$

In this case the rates of expansion of the jet boundaries in the plane of symmetry and in the lateral direction may be written in the form:

$$\delta_y' = r_c' - v_a/u^{(0)},\ \delta_z' = r_c' + v_a/u^{(0)}. \tag{2}$$

Subtracting the first of relations (2) from the second term by term, we obtain a relation between the rates of growth of the half-width of the jet in the plane of symmetry and in the lateral direction:

$$\delta_z' = \delta_y' + 2v_a/u^{(0)}. \tag{3}$$

As a result it may be assumed that the cross section of the jet will be elliptical rather than circular. The edges of the ellipse will be entrained by the flow, forming "horseshoes."

In accordance with the above, the additional flow rate ejected by the jet will be assumed to be proportional to the cross-stream velocity component normal to the jet trajectory and to the width of the jet in the lateral direction, with the same proportionality factor as in expression (1). Then the additional flow rate entering the jet on the interval from the nozzle exit to a given cross section x will be equal to

$$Q(x) = \varepsilon \int_0^x \delta_z V_{\infty n} dx. \tag{4}$$

If we assume that the jet cross section is elliptical, which corresponds to the experimental data on the initial length for not too large values of the cross stream/jet velocity ratio, and make use of the experimentally observed similarity of the velocity profiles in

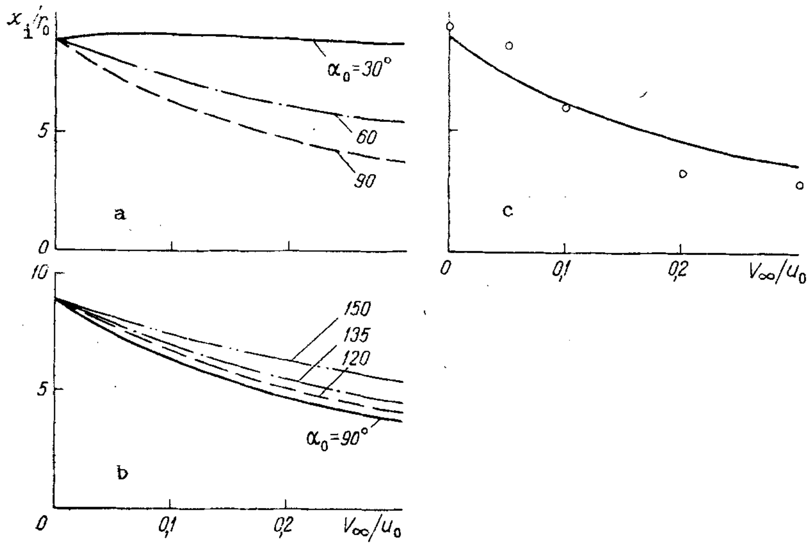

Fig. 2. Initial length as a function of the jet outflow angle and the cross stream-jet velocity ratio.

cross sections of the jet, we can solve the problem by the integral method and find the characteristics of the circular jet in the cross stream.

We note that the determination of the complex velocity and pressure field at the nozzle exit and the pressure distribution on the underlying surface around the jet is an essentially still unsolved problem which in [9, 10], where the Reynolds equations and the Navier–Stokes equations for three-dimensional jet flow in a cross stream were integrated numerically, led to a considerable discrepancy between the calculated and experimental velocity profiles. Accordingly, at present the use of the integral method for solving such a complex problem as that of a circular jet in a cross stream is more justifiable than the use of a numerical method, since in the first case the error associated with replacing the real, essentially nonuniform velocity and pressure field at the nozzle exit by a uniform field is smoothed out as a result of the fact that the velocity enters into the integral relations as part of the integrand. The errors introduced by replacing the real horseshoe-shaped jet cross section by an ellipse at relatively large distances from the nozzle exit and by using a universal curve for the velocity profile are similarly smoothed out.

Let us consider the solution of the problem of the initial length of a circular jet in a cross stream obtained by the integral method within the framework of the flow model described above.

Let the jet flow out at angle α_0 to the stream. Since near the nozzle exit it deviates only slightly from the initial direction, we may assume that the velocity component u_δ along the jet trajectory is constant and equal to

$$u_\delta = V_\infty \cos\alpha_0, \tag{5}$$

and that along the axis the pressure does not vary over the initial length. We note that the assumption should be the more nearly correct the smaller the ratio of the stream and jet velocities. Then on the initial length the excess momentum of the jet may be assumed to be constant. We will also assume that the cross section is elliptical, i.e., that the lines of constant velocity are ellipses. We will describe the profile of the longitudinal velocity component by means of Schlichting's expression for the initial length of a jet [11]

$$\frac{u-u_\delta}{u_0-u_\delta} = 2\eta^{3/2}-\eta^3;\quad \eta = \frac{r_1-r}{r_1-r_2}. \tag{6}$$

Figure 1 shows the velocity profile for a jet angle α_0 = 90° constructed in the dimensionless coordinates $u/u_0 = f(\eta_{exp})$, where

$$\eta_{exp} = \frac{r_{0,2}-r}{r_{0,2}-r_{0,95}}. \tag{7}$$

The curve in Fig. 1 is the Schlichting profile (6) replotted in coordinates (7). Clearly, the Schlichting equation quite satisfactorily describes the velocity profile on the initial length of a circular jet in a cross stream. However, it should be noted that at large values of the velocity ratio with distance from the nozzle exit the experimental points at the rear of the jet are observed to depart from the universal curve.

If we use a universal dependence to describe the velocity profile, then, as noted above, we can solve the problem by means of the integral method. As the equations for determining the four boundaries of the mixing zone — two in the plane of symmetry y_{1m} and y_{2m} and two in the lateral direction z_{1m} and z_{2m} — we can use the constant excess momentum condition

$$4\int_0^{\pi/2} d\varphi \int_0^{r_1(\varphi)} u(u-u_\delta)\,r\,dr = \text{const} \tag{8}$$

and the flow rate equation

$$4\int_0^{\pi/2} d\varphi \int_0^{r_1(\varphi)} u\,r\,dr = Q_0 + Q_{ej} + Q_{add} \tag{9}$$

As the third equation it is possible to employ the condition of lateral expansion of the jet (3), which for the initial length takes the form:

$$z'_{1m} = y'_{1m} + 2v_a/u_0. \tag{10}$$

The fourth relation is automatically obtained from the condition that the lines of constant velocity be ellipses:

$$\frac{z_{2m}}{z_{1m}} = \frac{y_{2m}}{y_{1m}}. \tag{11}$$

The flow rate ejected by a circular jet in a cocurrent flow is easily calculated by solving the problem of the initial length for such a jet using the integral method. It is found to be equal to

$$\frac{Q_{ej}}{r_0^2 u_0} = 0.18\pi c\,\frac{1-m_\delta}{1+m_\delta}\,x\left[1 + 3.244m_\delta + (0.349 + 0.103m_\delta - 0.811m_\delta^2)\,c\,\frac{1-m_\delta}{1+m_\delta}\,x\right], \tag{12}$$

where c is determined from the condition that the width of the submerged jet mixing zone, approximated by a linear dependence, conform to experiment ($c = 0.25$-0.3); $m_\delta = u_\delta/u_0$.

In accordance with (4), the additional mass flowing into the jet is equal to

$$\frac{Q_{add}}{r_0^2 u_0} = \varepsilon\,\frac{V_{\infty n}}{u_0}\int_0^x z_{1m}\,dx,\quad V_{\infty n} = V_\infty \sin\alpha_0. \tag{13}$$

Substituting relations (12) and (13) in (9), after manipulation we find

$$0.129(1-m_\delta)\,y_{2m}z_{2m} + (0.179 + 0.321m_\delta)\,y_{1m}z_{1m} + 0.192(1-m_\delta)\,y_{1m}z_{1m} = q(x), \tag{14}$$

$$q(x) = 0.5 + (0.09 + 0.292m_\delta)\,c\,\frac{1-m_\delta}{1+m_\delta}\,x + (0.0314 + 0.0093m_\delta - 0.073m_\delta^2)\left(\frac{1-m_\delta}{1+m_\delta}\right)^2 c^2x^2 + \frac{\varepsilon}{2\pi}\,\frac{V_{\infty n}}{u_0}\int_0^x z_{1m}\,dx. \tag{15}$$

In obtaining relation (14) we used the easily evaluated integrals

$$\int_0^{\pi/2} r_1^2\,d\varphi = \frac{\pi}{2}\,y_{1m}z_{1m};\quad \int_0^{\pi/2} r_1 r_2\,d\varphi = \frac{\pi}{2}\,y_{1m}z_{1m};\quad \int_0^{\pi/2} r_2^2\,d\varphi = \frac{\pi}{2}\,y_{2m}z_{2m}. \tag{16}$$

From Eq. (8), using (16), we have

$$(0.106 + 0.073m_\delta)y_{1m}z_{1m} + (0.204 - 0.012m_\delta)\,y_{1m}z_{2m} + (0.190 - 0.061m_\delta)\,y_{2m}z_{2m} = 0.5. \tag{17}$$

Integrating Eq. (9) with respect to x subject to the condition that $y_{1m} = z_{1m}$ when $x = 0$, we obtain the relation between the lateral expansion of the jet and the expansion in the plane of symmetry

$$z_{1m} = y_{1m} + 2\varepsilon\,\frac{V_\infty}{u_0}(\sin\alpha_0)\,x. \tag{18}$$

The initial length and the width of the mixing zone at the end of it can be found from (15) and (17) by setting $y_{2m} = z_{2m} = 0$. We then obtain

$$y_{1mi} z_{1mi} = \frac{0.5}{0.106 + 0.073m_\delta}; \quad q(x_i) = \frac{0.5(0.179 + 0.321m_\delta)}{0.106 + 0.073m_\delta}. \tag{19}$$

The integral on the right side of (15) can be evaluated by means of expressions (18) and (2), in accordance with which

$$y_{1m} = y_{1c} - \varepsilon \frac{V_{\infty n}}{u_0} x, \tag{20}$$

where

$$y_{1c} = 1 + (0.54 - 0.158m_\delta) c \frac{1 - m_\delta}{1 + m_\delta} x. \tag{21}$$

Then for the unknown integral we obtain the expression

$$\int_0^x z_{1m} dx = x + \left[(0.27 - 0.079m_\delta) c \frac{1 - m_\delta}{1 + m_\delta} + \frac{\varepsilon}{2} \frac{V_{\infty n}}{u_0}\right] x^2. \tag{22}$$

From relation (18) there follows

$$z_{1mi} = y_{1mi} + 2\varepsilon \frac{V_\infty}{u_0} x_i \sin \alpha_0. \tag{23}$$

In Fig. 2 we have plotted the dependence $x_i = f(V_\infty/u_0, \alpha_0)$ determined from the simultaneous solution of relations (14), (15), (19)-(23) for jet angles $\alpha_0 \leq 90°$. Clearly, as the ratio V_∞/u_0 increases, the initial length is considerably shortened. It is also clear that for the same V_∞/u_0 a decrease in the jet angle leads, as expected, to an increase in the initial length. An increase in the jet angle as compared with $\alpha_0 = 90°$ should lead to the jet developing in a counterflow near the nozzle exit. However, in this case (see [8]) the angle of thickening of the boundary layer does not depend on the velocity ratio and has the same value as for the submerged jet. Accordingly, in the expressions for calculating the characteristics of the initial length it is necessary to set $m_\delta = 0$. Obviously, the jet angle should then influence the jet characteristics only through the change in the cross stream velocity component normal to the jet trajectory. Since this decreases with increase in the jet angle, the additional ejection into the jet also decreases. Consequently, the initial length should increase, which is demonstrated in Fig. 2b, obtained as a result of calculating the initial length for jet angles $\alpha_0 \geq 90°$.

In Fig. 2c we have compared the calculated initial length with the experimental data [6, 7]. Clearly, the agreement is quite satisfactory.

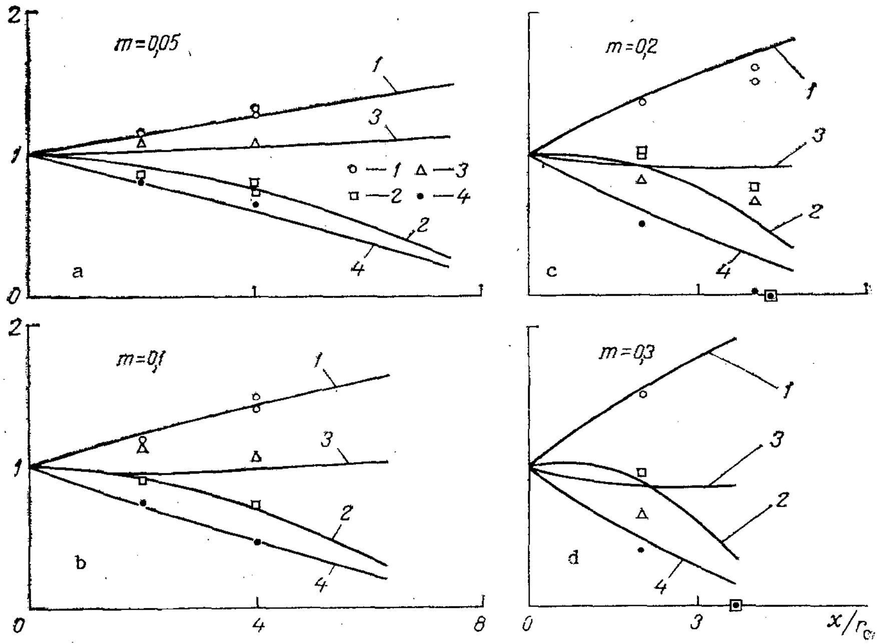

Fig. 3. Comparison of the results of calculation of the jet boundaries with experiment: 1) $z_{0.2}$; 2) $z_{0.95}$; 3) $y_{0.2}$; 4) $y_{0.95}$.

The system of equations (11), (14), (17), (18) is easily solved. Its solution yields the following expressions:

$$f = \frac{y_{2m}}{y_{1m}} = \frac{(b_1^2 + 4a_1c_1)^{1/2} - b_1}{2a_1}, \tag{24}$$

$$a_1 = (0.19 - 0.061m_\delta)\,q - 0.0645\,(1 - m_\delta), \tag{25}$$

$$b_1 = (0.204 - 0.012m_\delta)\,q - 0.096\,(1 - m_\delta),\ c_1 = 0.0895 + 0.16m_\delta - (0.106 + 0.073m_\delta)\,q,$$

$$y_{1m} = \left(\varepsilon^2 \frac{V_\infty^2}{u_0^2} x^2 \sin^2\alpha_0 + n\right)^{1/2} - \varepsilon \frac{V_\infty}{u_0} x \sin\alpha_0, \tag{26}$$

$$n = 0.5/[0.106 + 0.073m_\delta + (0.204 - 0.012m_\delta)\,f + (0.19 - 0.061m_\delta)\,f^2],$$

$$y_{2m} = y_{1m} f, \tag{27}$$

$$z_{1m} = y_{1m} + 2\varepsilon \frac{V_\infty}{u_0} x \sin\alpha_0,\ z_{2m} = z_{1m} f. \tag{28}$$

In Fig. 3 we have compared the calculated boundaries of the jet mixing zone in the plane of symmetry and the lateral direction obtained using expressions (24)-(28) for α_0 = 90° with the experimental data [6, 7]. The quantity ε was taken equal to 0.8 and was essentially a second empirical constant. Clearly, the agreement between the calculations and the experimental data is quite satisfactory.

Thus, it may be assumed that the proposed approach to the description of the flow on the initial length makes it possible to explain quantitatively as well as qualitatively the principal effects observed in experiments to study the interaction of a circular jet and a cross flow: shortening of the potential core with increase in the flow-jet velocity ratio, the barrel shape of the core in the lateral direction, contraction of the jet in the plane of symmetry and its more intense lateral expansion.

NOTATION

c, an empirical constant; $m_\delta = u_\delta/u_0$; Q, flow rate; Q_0, initial flow rate; Q_{ej}, flow rate ejected by the normal submerged jet; Q_{add}, additional flow rate ejected by the jet; r_0, radius of the jet at the nozzle exit; r_c, radius of the submerged jet; r_1 and r_2, radius vectors of the jet boundary and the constant total pressure core respectively; $r_{0.2}$ and $r_{0.95}$, radius vectors of the points at which the excess total pressure is equal to 0.2 and 0.95 of the excess total pressure in the potential core; r and ϕ, polar coordinates moving with the jet cross section; u_0, jet exit velocity; u, velocity component along the jet trajectory; $u^{(0)}$, characteristic velocity: $u^{(0)} = u_0$ on the initial length, $u^{(0)} = u_m$ in the main part of the jet; u_δ, velocity at the edge of the jet; V_∞, cross flow velocity; $V_{\infty n}$, cross flow velocity component normal to the jet trajectory; x, y, z, Cartesian coordinates moving with the jet trajectory: x along the jet axis, y perpendicular to x in the plane of symmetry of the jet, and z perpendicular to the plane xy; x_i, initial length; X, Y, Z, Cartesian coordinates tied to the nozzle: X along the cross flow, Y perpendicular to the cross flow, and Z perpendicular to the plane XY; y_{1m} and z_{1m}, boundaries of the jet in the plane of symmetry and the lateral direction respectively; y_{2m} and z_{2m} boundaries of the constant total pressure core in the plan of symmetry and the lateral direction respectively; y_{1c}, ordinate of the outer edge of the mixing zone of the submerged jet; α_0, jet expulsion angle; δ_y and δ_z, half-widths of the jet in the plane of symmetry and the lateral direction; ε, empirical constant.

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EFFECT OF BODY SHAPE ON THE CHARACTERISTICS OF A SELF-SIMILAR PLANE WAKE

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UDC 532.517.4

The turbulence characteristics in the wake of an elongated plane body with two sharp trailing edges have been experimentally investigated. It is shown that in the self-similar region the parameters of the plane wake depend on the body shape.

It is known [1] that for fairly large Reynolds numbers $Re = U_\infty d/\nu$ and at large distances x_1/d from the body various types of turbulent wakes develop self-similarly. This means that the statistical turbulence characteristics, which are determined by the large-scale components of the motion, can be represented in the form:

$$\Delta U = U_* f(x_2/l_*);\ \overline{u_i u_j} = U_*^2 g_{ij}(x_2/l_*) \text{ etc.} \tag{1}$$

Here, $\Delta U = U - U_\infty$ is the average longitudinal velocity defect in the wake; $\overline{u_i u_j}$ are the components of the Reynolds stress tensor. In (1) the characteristic scales of velocity U_* and length ℓ_* depend only on the longitudinal coordinate x_1, the free-stream velocity U_∞ and the drag $F = C_x \rho U_\infty^2 S/2$.

In the case of a plane wake U_* and ℓ_* are given by

$$U_* = U_\infty \left[\frac{C_x d}{2(x_1 + x_0)}\right]^{0,5};\ l_* = [0.5 C_x d (x_1 + x_0)]^{0,5}, \tag{2}$$

where x_0 is the virtual origin.

For a long time it was considered [1, 2] that in the region in which the flow in the wake is self-similar the statistical turbulence characteristics can be completely determined by specifying the drag, the free-stream flow velocity U_∞ and the position of the virtual origin x_0. In other words, the functions f, g_{ij}, etc. in Eqs. (1) were assumed to be universal, i.e., not to depend on the body shape. Then, in a series of experimental studies [3-5] it was shown that the structure of the axisymmetric wake depends not only on the drag and the free-stream velocity but also on the shape of the body. As far as the plane wake is concerned, the hypothesis of the universality of the characteristics in the self-similar flow region is so far considered to be correct [2]. Despite the fact that the hypothesis was partially confirmed in [6] by comparing the results of measurements in the wakes of circular and elliptical cylinders, there is reason to doubt its applicability to all plane bodies without exception. In particular, in [7] it was shown that in the wake of an elongated plane body with two sharp trailing edges the shape of the self-similar average longitudinal velocity profile differs significantly from the so-called universal profile observed in the wake

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 6, pp. 912-919, June, 1988. Original article submitted December 30, 1986.

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